

Kurvendiskussion

$$f(x) = x^2 \cdot e^{-x}$$

1.) Ableitungen

$$\begin{aligned} f'(x) &= 2x \cdot e^{-x} + x^2 \cdot (-1) \cdot e^{-x} \\ &= e^{-x}(2x + x^2 \cdot (-1)) \\ &= e^{-x}(2x - x^2) \end{aligned}$$

$$\begin{aligned} f''(x) &= (-1) \cdot e^{-x} \cdot (2x - x^2) + e^{-x}(2 - 2x) \\ &= e^{-x}(-2x + x^2 + 2 - 2x) \\ &= e^{-x}(x^2 - 4x + 2) \end{aligned}$$

$$\begin{aligned} f'''(x) &= (-1) \cdot e^{-x}(x^2 - 4x + 2) + e^{-x}(2x - 4) \\ &= e^{-x}(x^2 + 4x - 2 + 2x - 4) \\ &= e^{-x}(-x^2 + 6x - 6) \end{aligned}$$

2.) Nullstellen

$$f(x) = 0$$

$x^2 \cdot e^{-x} = 0$ Da $e^{-x} \neq 0$ ist, gilt:

$$x^2 = 0 \Rightarrow x = 0$$

$$N(0|0)$$

3.) Extremstelle

$$f'(x) = e^{-x} \cdot (2x - x^2) \quad \text{Da } e^{-x} \neq 0 \text{ ist, gilt:}$$

$$(2x - x^2) = 0$$

$$x(2 - x) = 0$$

1. Fall: $x = 0$

2. Fall: $2 - x = 0$

$$x = 2$$

$$f''(0) = e^{-0} (-4 \cdot 0 + 0^2 + 2) = 2 > 0 \Rightarrow \text{TP}$$

$$\underline{\text{y-Wert}}: f(0) = 0^2 \cdot e^{-0} = 0 \Rightarrow \text{TP } (0 | 0)$$

$$f''(2) = e^{-2} (4 \cdot 2 + 2^2 + 2)$$

$$f''(2) = e^{-2} \cdot (-2) = -0.27 < 0 \Rightarrow \text{HP}$$

$$\underline{\text{y-Wert}}: f(2) = 2^2 \cdot e^{-2} = 0,54 \Rightarrow \text{HP } (2 | 0,54)$$

4.) Wendepunkte:

$$f'''(x) = 0$$

$$e^{-x}(x^2 - 4x + 2) = 0 \quad \text{Da } e^{-x} \neq 0 \text{ ist, gilt:}$$

$$x^2 - 4x + 2 = 0$$

$$x_{1,2} = 2 \pm \sqrt{4 - 2}$$

$$x_{1,2} = 2 \pm \sqrt{2}$$

$$x_1 = 2 + \sqrt{2} = 3,41$$

$$x_2 = 2 - \sqrt{2} = 0,58$$

$$f'''(0,58) = e^{-0,58} \cdot (6 \cdot (-0,58) - (-0,58)^2 - 6)$$

$$f'''(0,58) = -1,57 \neq 0 \Rightarrow \text{Wendepunkt!}$$

$$\underline{\text{y - Wert}}: f(0,58) = 0,58^2 \cdot e^{-0,58}$$

$$f(0,58) = 0,19 \rightarrow W_1(0,58 | 0,19)$$

$$f'''(3,41) = e^{-3,41} \cdot (6 \cdot (3,41) - (3,41)^2 - 6)$$

$$f'''(3,41) = 0,09 \neq 0 \Rightarrow \text{Wendepunkt!}$$

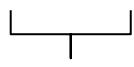
$$\underline{y\text{-Wert}}: f(3,41) = 3,41^2 \cdot e^{-3,41}$$

$$f(3,41) = 0,38 \Rightarrow W_2(3,41|0,38)$$

5.) Verhalten für $x \rightarrow +\infty$

$$\lim_{x \rightarrow +\infty} f(x) = \frac{x^2}{\cancel{x}} \cdot \frac{e^{-x}}{\cancel{x}}$$

$$+\infty \quad 0$$



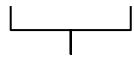
$$\rightarrow 0$$

$f(x)$ strebt gegen 0

Verhalten für $x \rightarrow -\infty$

$$\lim_{x \rightarrow -\infty} f(x) = \frac{x^2}{\cancel{x}} \cdot \frac{e^{-x}}{\cancel{x}}$$

$$+\infty \quad +\infty$$



$$\rightarrow +\infty$$

$f(x)$ strebt gegen $+\infty$

6.) Skizze

